

One-Time Offer - Test 1 ONLY!!!

Correct your test on separate paper. Same format (Page 1, Page 2, etc.).

I won't go looking for answers.

I won't give you points for page 2 stuff on page 1, ...

Anything that makes life harder for me, makes your grade lower.

Your Test 1 score will be the average of your 1st try and your 2nd try.

One question from test

⑧ (i) $(-3, 0), (3, 0), (0, 0)$
 (ii) $(0, 0)$

(b) $D = [-4, 4] = \{x \mid -4 \leq x \leq 4\}$
 $R = [-3, 3] = \{y \mid -3 \leq y \leq 3\}$

(c) (i) $(-4, -2) \cup (2, 4)$

~~(ii)~~ (ii) $(-2, 2)$

(d) (i) min of $y = -1$ @ $x = 2$
 (ii) max of $y = 1$ @ $x = -2$

$\sqrt{x^2 + 4} = x + 2$ No!!!

2.1 Properties of Linear Functions

PREPARING FOR THIS SECTION

- Lines (Foundations, Section F.3, pp. 18–26)
- Graphing Equations in Two Variables (Foundations, Section F.2, pp. 8–11)
- Linear Equations (Appendix A, Section A.8, pp. A60–A63)
- Functions (Section 1.1, pp. 40–51)
- The Graph of a Function (Section 1.2, pp. 54–58)
- Properties of Functions (Section 1.3, pp. 63–70)

Objectives:

- 1 Graph Linear Functions I will revisit the 1.5 techniques for these graphs.
- 2 Use Average Rate of Change to Identify Linear Functions
- 3 Determine Whether a Linear Function is Increasing, Decreasing, or Constant
- 4 Find the Zero of a Linear Function
- 5 Work with Applications of Linear Functions

'Are You Prepared?'

1. Graph $y = 2x - 3$. (p. 24)
2. Find the slope of the line joining the points $(2, 5)$ and $(-1, 3)$. (pp. 18–20)
3. List the intercepts of the equation $x^2 + 2y = 4$. (pp. 54–58)
4. Solve: $60x - 900 = -15x + 2850$. (pp. A61–A63)
5. If $f(x) = x^2 - 4$, find $f(-2)$. (pp. 44–46)
6. *True or False:* The graph of the function $f(x) = x^2$ is increasing on the interval $(0, \infty)$. (p. 78)

A **linear function** is a function of the form

$$y = mx + b$$

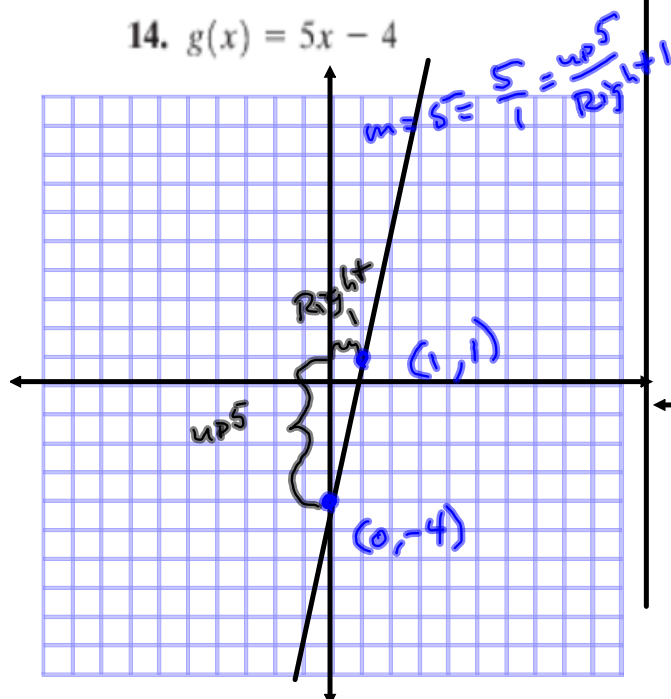
$$f(x) = mx + b$$

The graph of a linear function is a line with slope m and y-intercept b .

Let's look at a couple, graphing using

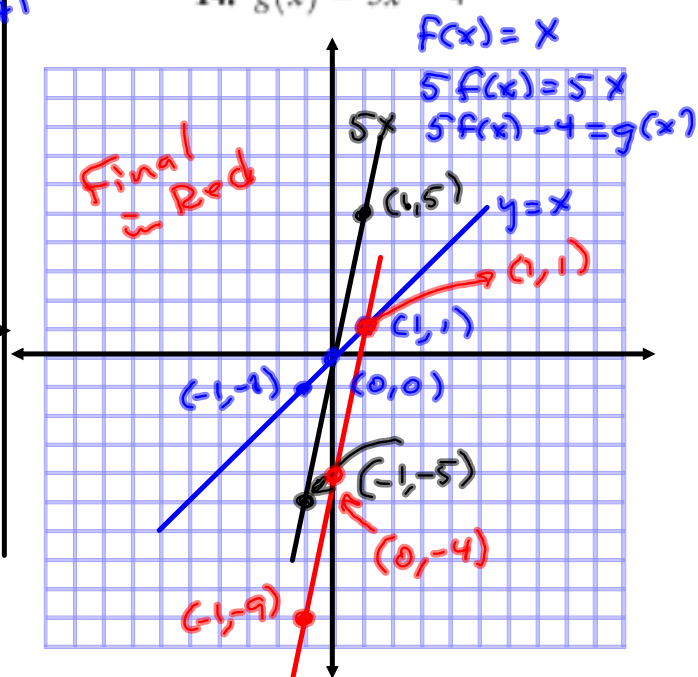
... the y-intercept and slope

14. $g(x) = 5x - 4$



... Section 1.5 transformations on the identity function $f(x) = x$.

14. $g(x) = 5x - 4$



The rate of change of a linear function is constant. That's *not* to say that a linear function is always a constant. But its slope (rate of change) *is* constant.

2 Use Average Rate of Change to Identify Linear Functions

Average Rate of Change of a Linear Function

Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is

$$\frac{\Delta y}{\Delta x} = m$$

27.

x	y = f(x)
-2	4
-1	1
0	-2
1	-5
2	-8

Notice $\Delta x = 1$ for all of these. So really, just checking Δy suffices.

$$\begin{array}{l} \Delta y = -3 \\ \Delta y = -3 \\ \Delta y = -3 \\ \Delta y = -3 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 4}{-1 - (-2)} = \frac{-3}{1} = -3$$

Necessary if the Δx 's aren't all the same.

$$\frac{\Delta y}{\Delta x} = \frac{-3}{1} \quad \checkmark \text{ pair of points}$$

Yes. Linear.

28.

x	y = f(x)
-2	1/4
-1	1/2
0	1
1	2
2	4

$$\Delta x = 1$$

$$\Delta x = 1$$

$$\Delta x = 1$$

$$\Delta x = 1$$

$$\Delta y = y_2 - y_1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\Delta y = \frac{1}{2}$$

Not linear.

3 Determine Whether a Linear Function Is Increasing, Decreasing or Constant

Increasing, Decreasing, and Constant Linear Functions

A linear function $f(x) = mx + b$ is increasing over its domain if its slope, m , is positive. It is decreasing over its domain if its slope, m , is negative. It is constant over its domain if its slope, m , is zero.

Increasing

$$f(x) = 17x - 11$$

Decreasing

$$f(x) = -0.005x + 5700$$

Constant

$$f(x) = 7$$

Special Case $m = 0$
Horizontal line $y = 7$
"Zero Slope"

$x = 5$ Not a function
Vertical Line "No Slope"
"undefined Slope"

4 Find the zeros of a linear function.

In Problems 21–26, (a) find the zero of each linear function

(b) graph each function using the zero and y-intercept.

22. $g(x) = 3x + 12$

Zeros of $g(x)$ found
by solving $g(x) = 0$
The x-values you obtain
are the zeros of $g(x)$.

$$g(x) = 0$$

$$3x + 12 = 0$$

$$3x = -12$$

$$x = -\frac{12}{3} = -4 \text{ is the zero}$$

$(-4, 0)$ is the x-intercept.

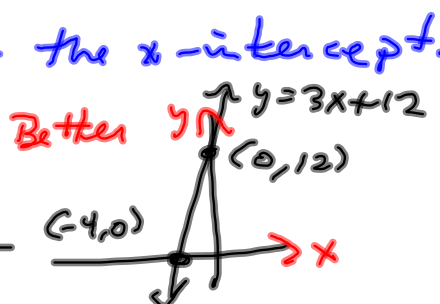
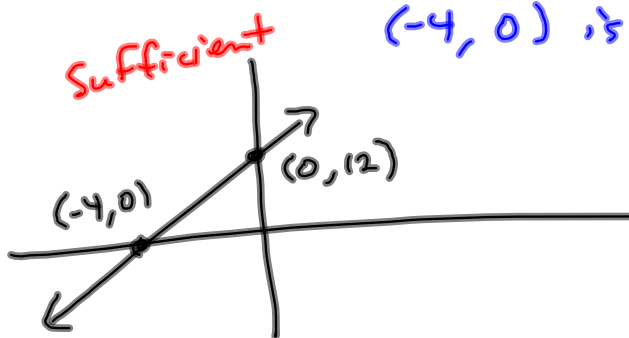
$(0, 12)$ is y-int.

This is called the "Intercept Method"

Keep this in mind for when we do a
different form of linear equations:

$$Ax + By = C$$

This general form of a linear equation
is *slick* for graphing by the intercept
method.



5 Work with Applications of Linear Functions

Cost and Revenue - Breakeven

(Linear) Supply and Demand Equations - Equilibrium

Other...

$$R = p \cdot x$$

$$\text{Revenue} = (\text{price})(\# \text{ sold})$$

$$p = \text{price } (\$)$$

$$x = \# \text{ of items sold}$$

48. **Supply and Demand** Suppose that the quantity supplied S and quantity demanded D of hot dogs at a baseball game are given by the following functions:

$$S(p) = -2000 + 3000p$$

$$D(p) = 10,000 - 1000p$$

Equilibrium:
 $S(p) = D(p)$

where p is the price.

- (a) Find the equilibrium price for hot dogs at the baseball game. What is the equilibrium quantity?
 (b) Determine the prices for which quantity demanded is less than quantity supplied.
 (c) What do you think will eventually happen to the price of hot dogs if quantity demanded is less than quantity supplied?

→ (a) $S(p) = D(p)$

$$\begin{array}{r} -2000 + 3000p = 10000 - 1000p \\ +2000 \qquad \qquad \qquad = +2000 \end{array}$$

$$3000p = 12000 - 1000p$$

$$1000p = \qquad \qquad \qquad +1000p$$

$$4000p = 12000$$

$$p = \frac{12000}{4000} = 3 \text{ is Equilibrium Price.}$$

(b) Want $D(p) < S(p)$

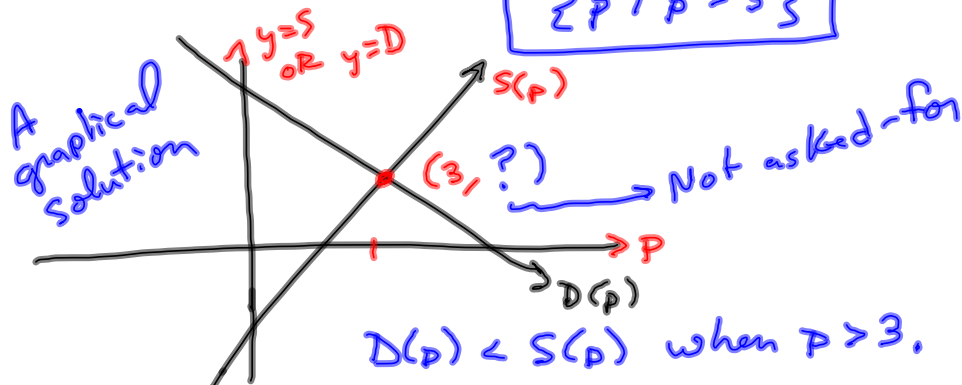
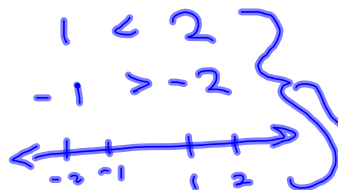
$$D(p) = 10000 - 1000p < -2000 + 3000p = S(p)$$

$$-1000p < -12000 + 3000p$$

$$-4000p < -12000$$

$$p > 3$$

$$\{p \mid p > 3\}$$



The point at which a company's profits equal zero is called the company's **break-even point**. For Problems 51 and 52, let R represent a company's revenue, let C represent the company's costs, and let x represent the number of units produced and sold each day.

- (a) Find the firm's break-even point; that is, find x so that $R = C$.
- (b) Find the values of x such that $R(x) > C(x)$. This represents the number of units that the company must sell to earn a profit.

Contemporary Cooks!

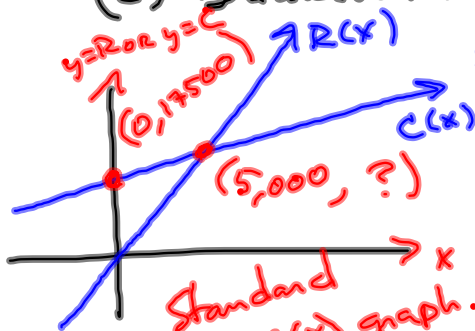
51. $R(x) = 8x$

$C(x) = 4.5x + 17,500$

Fixed Cost

Marginal Cost. Cost/item

(2) Break-even: $R(x) = C(x)$



$$8x = 4.5x + 17,500$$

$$3.5x = 17,500$$

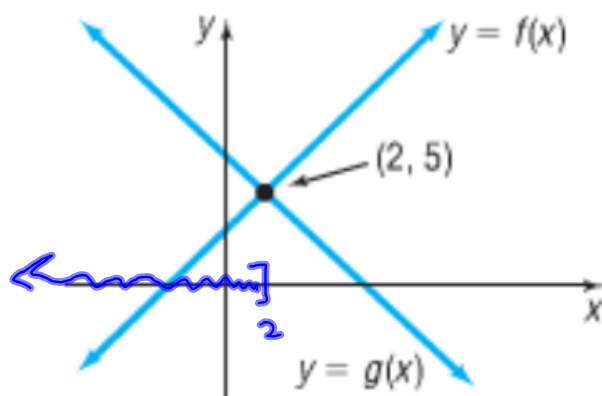
$$x = \frac{17,500}{3.5} \approx 5,000 \text{ items}$$

Standard $R(x), C(x)$ graph.
(b)

$R(x) > C(x)$ when $x > 5,000$.
(See picture. OR solve $R(x) > C(x)$)

Visual solutions of equations and inequalities:

40. In parts (a) and (b), use the following figure.



- (a) Solve the equation: $f(x) = g(x)$. $\Rightarrow x = 2$
(b) Solve the inequality: $f(x) \leq g(x)$. $\Rightarrow x \leq 2$